



东南大学 信息科学与工程学院
SCHOOL OF INFORMATION SCIENCE AND ENGINEERING

Active RIS Aided ISAC Systems: Beamforming Design and Performance Analysis

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RADIO



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Active RIS Aided ISAC Systems



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1 Background

2 Beamforming Design

3 Performance Analysis

4 Conclusions

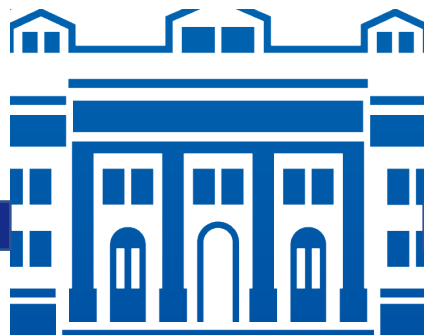


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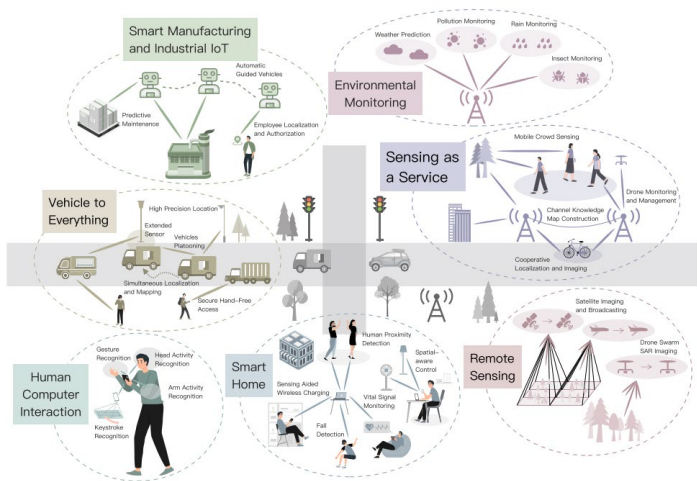
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Background

RIS aided ISAC



Integrated Sensing and Communication (ISAC)



Core advantages

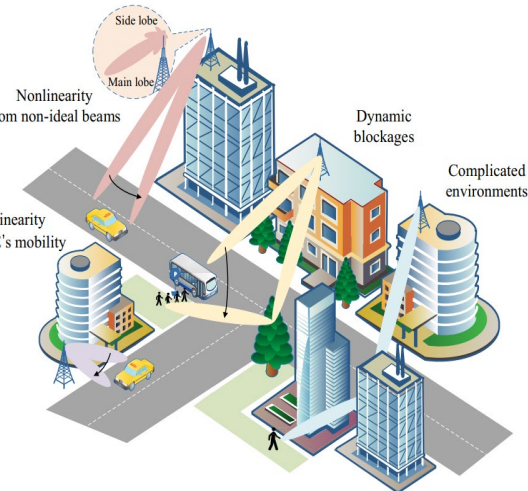
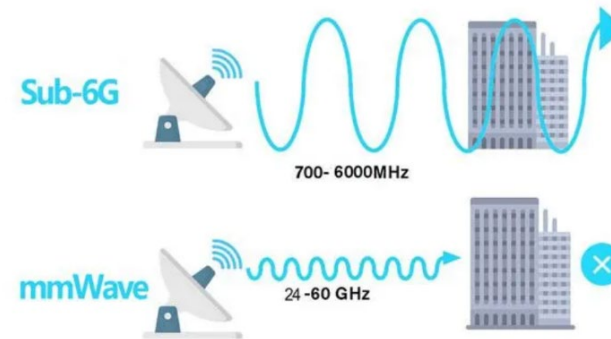
1. **Simultaneously** transmitting information while sensing the environment
2. Utilizing the **same** set equipment and frequency for both communication and sensing



Key Technology

Millimeter waves are considered the key frequency band for ISAC

- Abundant spectrum resources
- High data transmission rates and superior sensing precision

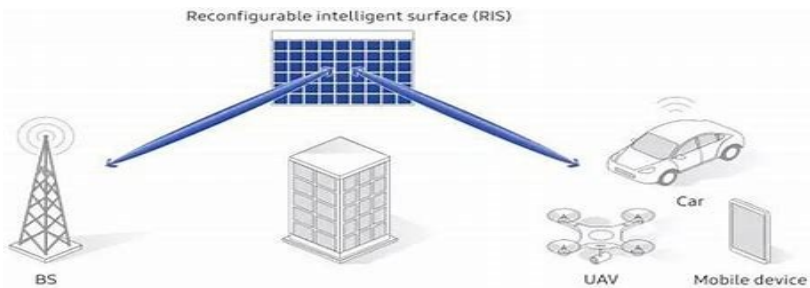


Research Bottleneck

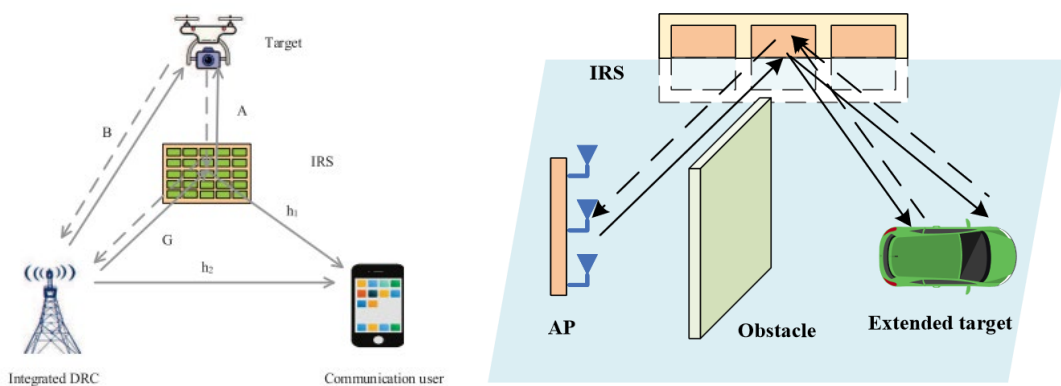
- ✓ Poor diffraction capability, **susceptible to obstruction.**
- ✓ Sensing services usually rely on the direct path.

Reconfigurable Intelligent Surface (RIS)

RIS can establish a **reliable virtual link**, bypassing obstacles.



RIS aided ISAC systems



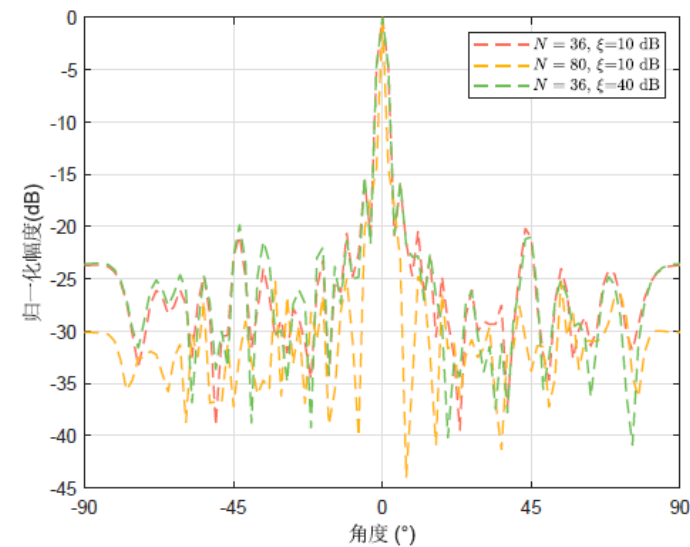
Demo Scenario

Maximize the beampattern gain at given angle θ

$$\begin{aligned} \mathcal{P}(\theta) &= \mathbb{E}(|\mathbf{a}^H(\theta) \mathbf{U} \mathbf{G} \mathbf{x}|^2) \\ &= \mathbf{a}^H(\theta) \mathbf{U} \mathbf{G} \mathbf{W} \mathbf{W}^H \mathbf{G}^H \mathbf{U}^H \mathbf{a}(\theta) \end{aligned}$$

Conclusion:

RIS is able to generate **high resolution** beam towards the target.

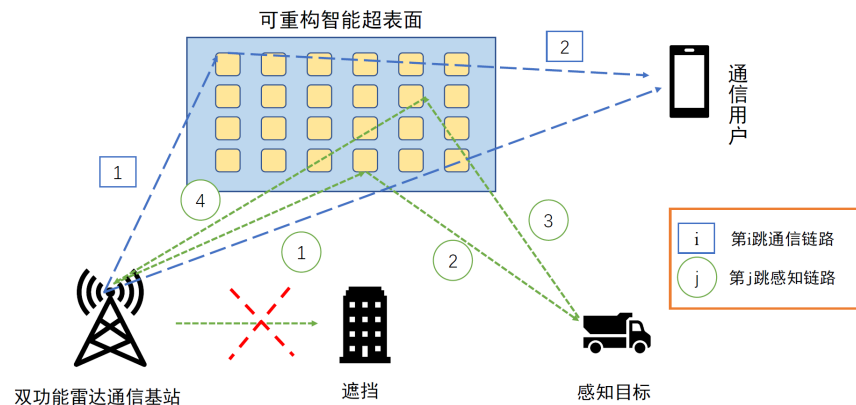
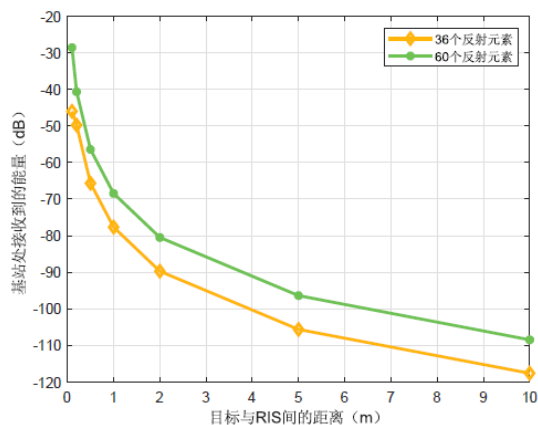


Passive RIS Assisted ISAC Systems: A Practical Problem

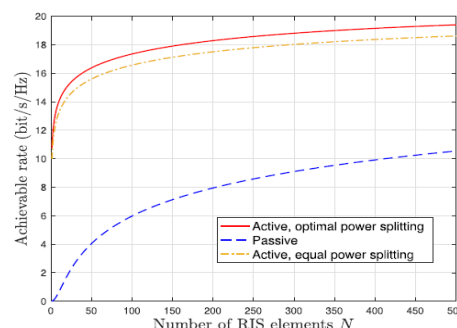
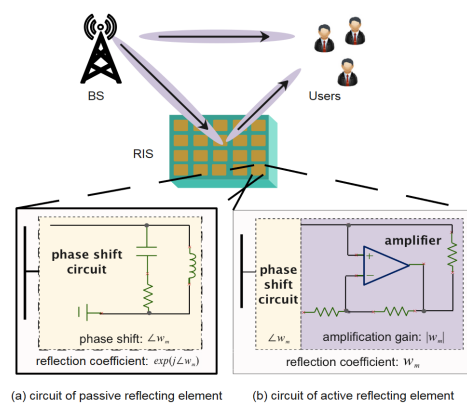
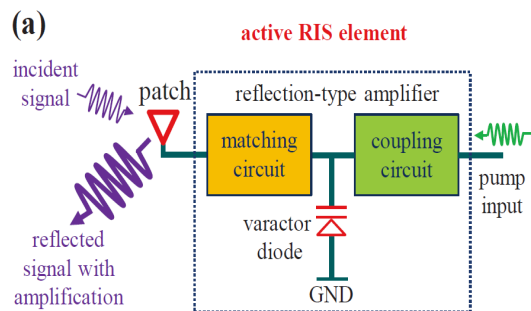
High Resolution but **Low SNR** ← Multiplicative Fading

Signals reflected off the RIS/Target mostly dissipate energy, resulting in an **equivalent channel path loss that is a product of individual path losses** rather than their sum, i.e., $\beta_{all} \approx \beta_1\beta_2\beta_3\beta_4\dots$

- Assuming the transmit power of BS 1W and the target located 10m away from the RIS, the received power at the BS is approximately **-80dBm**.



Solution: Active RIS

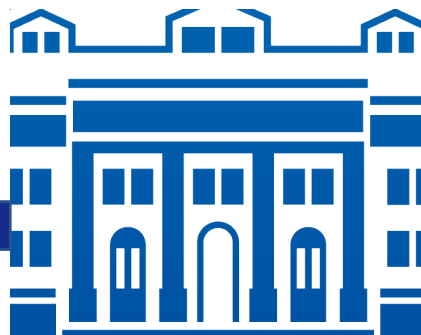


- Outperform the passive RIS in communication systems.
- In active RIS- ISAC systems, the signal can be amplified twice, countering multiplicative fading.

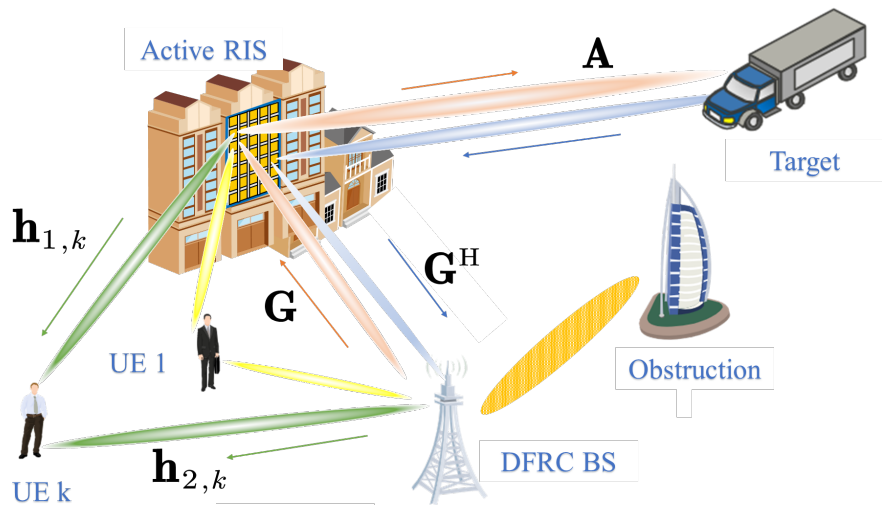
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Beamforming Design

Active RIS aided ISAC Systems



Active RIS aided ISAC systems



Communication Model

The received signal of the k -th UE:

$$y_{c,k} = \mathbf{h}_{1,k}^H \mathbf{U} \mathbf{G} \mathbf{x} + \mathbf{h}_{1,k}^H \mathbf{U} \mathbf{v}_1 + \mathbf{h}_{2,k}^H \mathbf{x} + z_k$$

The SINR of the k -th UE

$$\text{SINR}_k = \frac{\mathbf{h}_k^H \mathbf{R}_k \mathbf{h}_k}{\mathbf{h}_k^H (\mathbf{R} - \mathbf{R}_k) \mathbf{h}_k + \sigma^2 \mathbf{h}_{1,k}^H \mathbf{U} \mathbf{U}^H \mathbf{h}_{1,k} + \sigma_z^2}$$

Active RIS Model

The first received signal:

$$\mathbf{y}_1^r = \mathbf{U} \mathbf{G} \mathbf{x} + \mathbf{U} \mathbf{v}_1$$

Transmit power constraint:

$$\mathbb{E} [\|\mathbf{y}_1^r\|_2^2 + \|\mathbf{y}_2^r\|_2^2] \leq P_{RIS}$$

$\mathbf{v}_1 \ \mathbf{v}_2$: The induced noise by the active RIS

The second received signal:

$$\mathbf{y}_2^r = \mathbf{U}^H \mathbf{A} \mathbf{U} \mathbf{G} \mathbf{x} + \mathbf{U}^H \mathbf{A} \mathbf{U} \mathbf{v}_1 + \mathbf{U}^H \mathbf{v}_2$$

Power Amplification Gain:

$$|v_n|^2 \leq a_{RIS}, n = 1, \dots, N$$

Sensing Model

The received signal at the BS:

$$\mathbf{y}_r = \underbrace{\mathbf{G}^H \Phi^H \mathbf{A} \Phi \mathbf{G} \mathbf{x}}_{\text{Desired Signal}} + \underbrace{\mathbf{G}^H \Phi^H \mathbf{A} \Phi \mathbf{v}_1}_{\text{Interference Signal}} + \underbrace{\mathbf{G}^H \Phi \mathbf{v}_1 + \mathbf{G}^H \Phi^H \mathbf{v}_2}_{\text{Noise Induced by the Active RIS}} + \underbrace{\eta \mathbf{G}^H \Phi \mathbf{G} \mathbf{x} + \mathbf{z}_r}_{\text{Interference Plus Noise Matrix}}$$

- Desired Signal
- Interference Signal
- Noise Induced by the Active RIS

$$\text{SINR}_r = \text{Tr}(\mathbf{B} \mathbf{R} \mathbf{B}^H \mathbf{J}^{-1})$$

Radar SINR, which is closely related to the KL divergence in the context of hypothesis testing with two alternative hypotheses [1].

$$\begin{aligned} \mathbf{B} &= \mathbf{G}^H \mathbf{U}^H \mathbf{A} \mathbf{U} \mathbf{G} \\ \mathbf{J} &= \mathbf{D} + \mathbf{E} \mathbf{R} \mathbf{E}^H \\ \mathbf{D} &= \sigma^2 \mathbf{C} \mathbf{C}^H + \sigma^2 \mathbf{G}^H \mathbf{U}^H \mathbf{U} \mathbf{G} + \sigma_r^2 \mathbf{I}_M \\ \mathbf{E} &= \eta \mathbf{G}^H \mathbf{U} \mathbf{G} \end{aligned}$$

Interference Plus Noise Matrix

Problem Formulation and Proposed Solver

Problem Formulation

$$\begin{aligned} \max_{\mathbf{W}, \mathbf{U}} \quad & \text{SINR}_r \\ \text{s.t.} \quad & \text{SINR}_k \geq \xi, \quad k = 1, \dots, K, \\ & \text{Tr}(\mathbf{W}\mathbf{W}^H) \leq P_{BS}, \\ & \mathbb{E}[\|\mathbf{y}_1^r\|_2^2 + \|\mathbf{y}_2^r\|_2^2] \leq P_{RIS}, \\ & |v_n|^2 \leq a_{RIS}, \quad n = 1, \dots, N, \end{aligned}$$

Difficulties of Solving this Problem

- Coupled active RIS reflecting coefficients and BS beamforming matrices
- Radar SINR involves an **inversion** operation, and exhibits non-convex characteristics
- **Quartic function** induced by the **multiple self reflection** at the active RIS

Challenge 1: The inverse in the Radar SINR



Iteratively optimizes the Radar SINR by MM algorithm



$$\begin{aligned} \text{SINR}_r &= \text{Tr}(\mathbf{B}\mathbf{R}\mathbf{B}^H \mathbf{J}^{-1}) \\ \mathbf{J} &= \mathbf{D} + \mathbf{E}\mathbf{R}\mathbf{E}^H \end{aligned}$$

Define $\mathbf{X} = \mathbf{B}\mathbf{W}$

$\text{Tr}(\mathbf{X}^H \mathbf{J}^{-1} \mathbf{X})$ is jointly convex w.r.t. \mathbf{X}, \mathbf{J}

$$\begin{aligned} \text{Tr}(\mathbf{X}^H \mathbf{J}^{-1} \mathbf{X}) &\geq 2\text{Re}(\text{Tr}(\mathbf{X}_i^H \mathbf{J}_i^{-1} \mathbf{X})) \\ &\quad - \text{Tr}(\mathbf{J}_i^{-1} \mathbf{X}_i \mathbf{X}_i^H \mathbf{J}_i^{-1} \mathbf{J}) \end{aligned}$$

According to [42], we obtain the following first-order derivatives:

$$\frac{\partial}{\partial \mathbf{X}} \text{Tr}(\mathbf{J}^{-1} \mathbf{X} \mathbf{X}^H) = \mathbf{J}^{-T} \mathbf{X}^*, \quad (\text{A.2})$$

$$\frac{\partial}{\partial \mathbf{X}^*} \text{Tr}(\mathbf{X} \mathbf{X}^H \mathbf{J}^{-1}) = \mathbf{J}^{-1} \mathbf{X}, \quad (\text{A.3})$$

$$\frac{\partial}{\partial \mathbf{J}} \text{Tr}(\mathbf{X}^H \mathbf{J}^{-1} \mathbf{X}) = -(\mathbf{J}^{-1} \mathbf{X} \mathbf{X}^H \mathbf{J}^{-1})^T. \quad (\text{A.4})$$

By substituting (A.2)-(A.4) into (A.1), the lower bound can be obtained as

$$\begin{aligned} \text{Tr}(\mathbf{X}^H \mathbf{J}^{-1} \mathbf{X}) &\geq 2\text{Re}(\text{Tr}(\mathbf{X}_i^H \mathbf{J}_i^{-1} \mathbf{X})) \\ &\quad - \text{Tr}(\mathbf{J}_i^{-1} \mathbf{X}_i \mathbf{X}_i^H \mathbf{J}_i^{-1} \mathbf{J}). \end{aligned} \quad (\text{A.5})$$

By using the above derivations, we prove that the surrogate function satisfies the lower bound Condition 3 in Section IV-A, and also satisfies Condition 1 and Condition 2. Hence, the proof is completed.

MM (Majorization-Minimization)

- Majorization: Build an auxiliary function to replace the original one, ensuring it is equal to or greater than the original function at the current optimization point.
- Minimization: Optimize the auxiliary function to obtain a solution that approximates the original function.

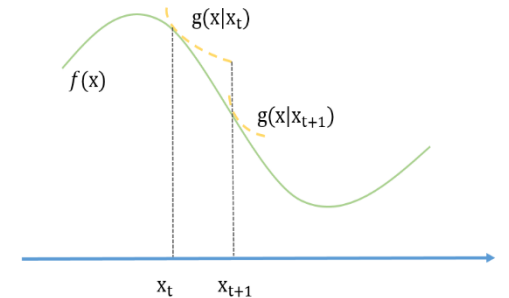


Illustration of the MM algorithm

Proposed Solver

- Optimize the BS beamforming matrix \mathbf{W}

$$\max_{\mathbf{W}} f(\mathbf{W})$$

$$\text{s.t. } (1 + \xi^{-1})(2\text{Re}(\mathbf{h}_k^H \mathbf{w}_{M+k,i} \mathbf{w}_{M+k,i}^H \mathbf{h}_k) - \mathbf{h}_k^H \mathbf{w}_{M+k,i} \mathbf{w}_{M+k,i}^H \mathbf{h}_k) \geq \mathbf{h}_k^H \mathbf{R} \mathbf{h}_k + d_k, \quad k = 1, 2, \dots, K,$$

$$\text{Tr}(\mathbf{W} \mathbf{W}^H) \leq P_{BS},$$

$$\|\mathbf{U}^H \mathbf{A} \mathbf{U} \mathbf{G} \mathbf{W}\|_F^2 + \|\mathbf{U} \mathbf{G} \mathbf{W}\|_F^2 \leq e.$$



Transmit power constraint of the active RIS

Communication SINR, expressed as the difference of convex functions, is resolved using SCA.

$$f(\mathbf{W}) = 2\text{Re}(\text{Tr}(\mathbf{W}_i^H \mathbf{B}^H \mathbf{J}_i^{-1} \mathbf{B} \mathbf{W})) - \text{Tr}(\mathbf{J}_i^{-1} \mathbf{B} \mathbf{R}_i \mathbf{B}^H \mathbf{J}_i^{-1} \mathbf{J})$$

The original problem is transformed into a **convex problem**, which can be directly solved.

Initial point optimization for MM algorithm

Achieving a high-performance, strictly feasible initial point is facilitated by SDR method:

By denoting the feasible solution of the relaxed problem $\tilde{\mathbf{R}}, \tilde{\mathbf{R}}_k$, we can formulate the feasible solution of the original problem as

$$\hat{\mathbf{w}}_{c,k} = \frac{\tilde{\mathbf{R}}_k \mathbf{h}_k}{\sqrt{\mathbf{h}_k^H \tilde{\mathbf{R}}_k \mathbf{h}_k}} \quad \hat{\mathbf{R}}_k = \hat{\mathbf{w}}_{c,k} \hat{\mathbf{w}}_{c,k}^H$$

$$\hat{\mathbf{W}}_r \hat{\mathbf{W}}_r^H = \tilde{\mathbf{R}} - \sum_{k=1}^K \hat{\mathbf{R}}_k \quad \leftarrow \text{Cholesky Decomposition}$$

Lemma 2

Provide a sufficient condition for feasibility in the original problem based ZF precoding: IF there exists ρ satisfying

$$\rho^4 \|\mathbf{A} \mathbf{G} \mathbf{W}^*\|_F^2 + \rho^2 \|\mathbf{G} \mathbf{W}^*\|_F^2 + 2\rho^2 \sigma^2 + \rho^4 \sigma^2 \text{Tr}(\mathbf{A} \mathbf{A}^H) \leq P_{RIS},$$

$$\text{Rank}(\tilde{\mathbf{H}}) = K, \quad 1 \leq \rho \leq \sqrt{a_{RIS}}, \quad \text{Tr}(\text{Diag}(\tilde{d}_1, \dots, \tilde{d}_K)(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}})^{-1}) \leq \frac{P_{BS}}{\xi}$$

Optimize the RIS reflecting coefficient matrix \mathbf{U}

The optimization of the **quartic function**

The lower bound of the Radar SINR:

$$\begin{aligned} \text{SINR}_r &= \text{Tr}(\mathbf{B}\mathbf{R}\mathbf{B}^H \mathbf{J}^{-1}) \\ &\geq 2\text{Re}(\text{Tr}(\mathbf{B}\mathbf{R}\mathbf{B}_i^H \mathbf{J}_i^{-1})) - \text{Tr}(\mathbf{J}_i^{-1} \mathbf{B}_i \mathbf{R} \mathbf{B}_i^H \mathbf{J}_i^{-1} \mathbf{J}) \\ \text{Tr}(\mathbf{J}_i^{-1} \mathbf{B}_i \mathbf{R} \mathbf{B}_i^H \mathbf{J}_i^{-1} \mathbf{J}) &= \text{Tr}(\mathbf{T}_i \mathbf{J}) \\ &= \underbrace{\sigma^2 \text{Tr}(\mathbf{T}_i \mathbf{G}^H \mathbf{U}^H \mathbf{A} \mathbf{U} \mathbf{U}^H \mathbf{A}^H \mathbf{U} \mathbf{G})}_{\text{Cubic Terms}} + 2\sigma^2 \text{Re}(\text{Tr}(\mathbf{T}_i \mathbf{G}^H \mathbf{U} \mathbf{U}^H \mathbf{A} \mathbf{U} \mathbf{G})) + a. \end{aligned}$$

Other Terms



The transmit power of the active RIS:

$$\begin{aligned} &\text{Tr}(\underbrace{\mathbf{U}^H \mathbf{A} \mathbf{U} \mathbf{G} \mathbf{R} \mathbf{G}^H \mathbf{U}^H \mathbf{A}^H \mathbf{U}}_{\text{Quartic Function}}) + \sigma^2 \text{Tr}(\mathbf{U}^H \mathbf{A} \mathbf{U} \mathbf{U}^H \mathbf{A}^H \mathbf{U}) \\ &+ \text{Tr}(\mathbf{U} \mathbf{G} \mathbf{R} \mathbf{G}^H \mathbf{U}^H) + 2\sigma^2 \text{Tr}(\mathbf{U} \mathbf{U}^H) \leq P_{RIS}. \end{aligned}$$

Hilbert's 17th Problem

? Solving **multi-dimensional high-order optimization problems**, **P or NP?**
Some of these problems are NP-hard.



A When the optimization problem can be expressed in the form of a **sum of squares (SOS)**, it can be solved through a class of SDP algorithms (**P**) [2].

[2] Lasserre J B. An introduction to polynomial and semi-algebraic optimization[M]. Cambridge University Press, 2015.

Algorithm Development

Joint Beamforming Design

Algorithm 1 Joint beamforming for the active RIS-aided ISAC system

Input: The maximum iteration time for the beamforming matrix optimization t_1^{\max} , the maximum iteration time for the reflecting coefficient matrix optimization t_2^{\max} , the maximum iteration time for the AO t^{\max} .

- 1: Set $t = 0$, generate a random reflecting coefficient \mathbf{v}^0 .
- 2: Drop the rank-one constraint and obtain a feasible solution of the relaxed Problem (24) $\hat{\mathbf{R}}_k$ and $\hat{\mathbf{R}}$ via CVX.
- 3: Calculate the beamforming vector $\hat{\mathbf{w}}_{c,k}$ and matrix $\hat{\mathbf{W}}_r$ according to (25) and (26), and set $\mathbf{W}^0 = [\hat{\mathbf{W}}_r, \hat{\mathbf{w}}_{c,1}, \hat{\mathbf{w}}_{c,2}, \dots, \hat{\mathbf{w}}_{c,K}]$.
- 4: **Repeat**
- 5: Set $t_1 = 0$, and $\mathbf{W}_0 = \mathbf{W}^t$.
- 6: **Repeat**
- 7: Obtain \mathbf{W}_{t_1+1} by solving (23) with given \mathbf{W}_{t_1} .
- 8: $t_1 = t_1 + 1$.
- 9: **Until** $t_1 = t_1^{\max}$
- 10: Set $t_2 = 0$, and $\mathbf{v}_0 = \mathbf{v}^t$.
- 11: **Repeat**
- 12: Obtain \mathbf{v}_{t_2+1} by solving Problem (53) with fixed \mathbf{v}_{t_2} .
- 13: $t_2 = t_2 + 1$.
- 14: **Until** $t_2 = t_2^{\max}$
- 15: $\mathbf{W}^t = \mathbf{W}_{t_1}$, $\mathbf{v}^t = \mathbf{v}_{t_2}$, and $t = t + 1$.
- 16: **Until** $t = t^{\max}$

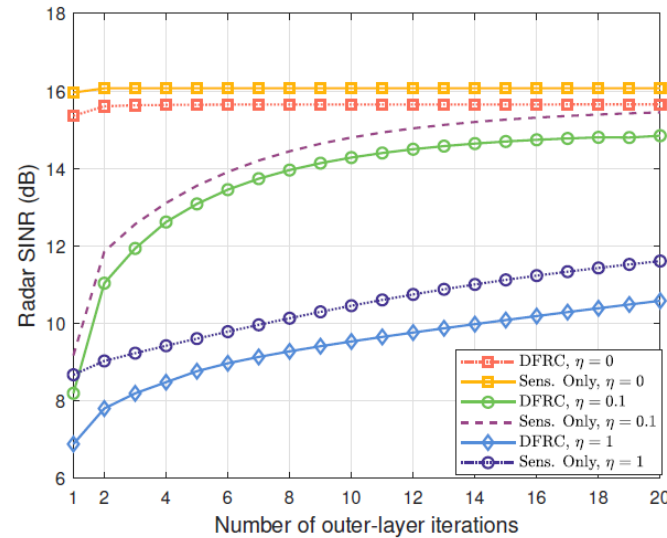


Fig. 3: Convergence behaviors.

- Fast convergence
- Determined by the SI cancellation coefficients

High
Computation
Complexity

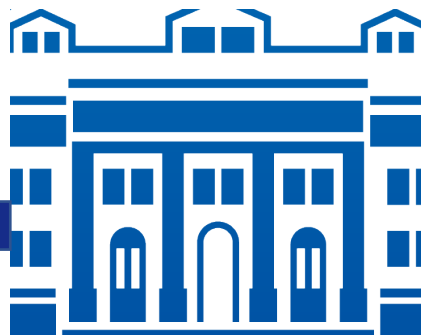
The computation complexity of optimizing the BS beamforming matrix is $o_f = \mathcal{O}(K^{3.5}N^6)$

The computation complexity of optimizing the RIS reflecting coefficients is $o_e = \mathcal{O}(K^{1.5}N^6)$

3

Performance Analysis

Active RIS aided ISAC Systems





Performance Analysis of the Active RIS aided Sensing Systems

Basic idea: How **the key parameters of this system** influence the sensing performance.

- ❑ Theoretical Problem: How much gain can we get when the number of the reflecting elements is **extremely large**?
- ❑ Practical Problem 1: What is the optimal **transmit power allocation** between the active RIS and the BS?
- ❑ Practical Problem 2: **Where to deploy** the active RIS to achieve higher sensing SINR?

Core work:
Approximate
sensing SINR,
i.e.,



$$\begin{matrix} \text{SINR}_{\text{up}} & \text{SINR}_{\text{low}} \\ \text{SINR}_{\text{app}} & \text{SNR} \end{matrix}$$

Ideal Assumption:

- Sensing-only BS with single antenna
- One amplifier is equipped on the active RIS
- The noise power is omitted in the transmit power of the active RIS

Simplified Problem: The interplay between the beamforming gain f^2 and the power amplification gain ρ

$$\begin{aligned} \max_{\rho, f^2} & \frac{P_t \rho^4 f^4}{\sigma_r^2 + \rho^4 \sigma^2 f^2 N \beta_r + 2\rho^2 \sigma^2 N g^2 + g_1} \\ \text{s.t.} & \rho^2 \leq a_{\text{RIS}}, \\ & P_t \rho^4 f^2 N \beta_r + P_t \rho^2 N g^2 \leq P_{\text{RIS}}. \end{aligned}$$

Scaling Order Derivation with Infinity Large Number of Reflecting Elements

Scaling order of the RIS-aided communication systems [3]

- The SINR has a scaling order of N^2 due to noise free nature of the RIS.
- For MIMO, a transmit/received beamforming gain of M can be achieved.

Scaling order of the active RIS-aided communication systems [4]

- ◆ The SINR has a scaling order of N considering the induced noise at the active RIS.
- ◆ In most case, the active RIS outperforms passive RIS due to the power amplification gain.

Scaling order of the active RIS-aided sensing systems

Theorem 1: For the active RIS-aided sensing systems, the radar SINR increases with the number of reflecting elements N as $N \rightarrow \infty$ according to

$$\lim_{N \rightarrow \infty} \frac{\log_2 \text{SINR}}{\log_2 N} = 1. \quad (63)$$

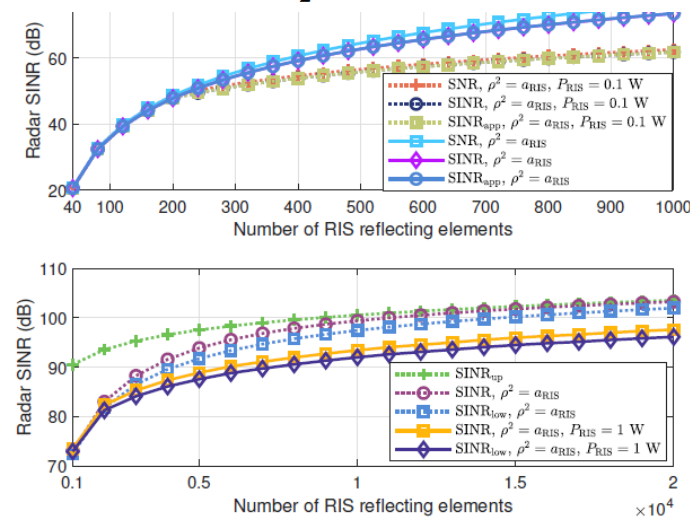
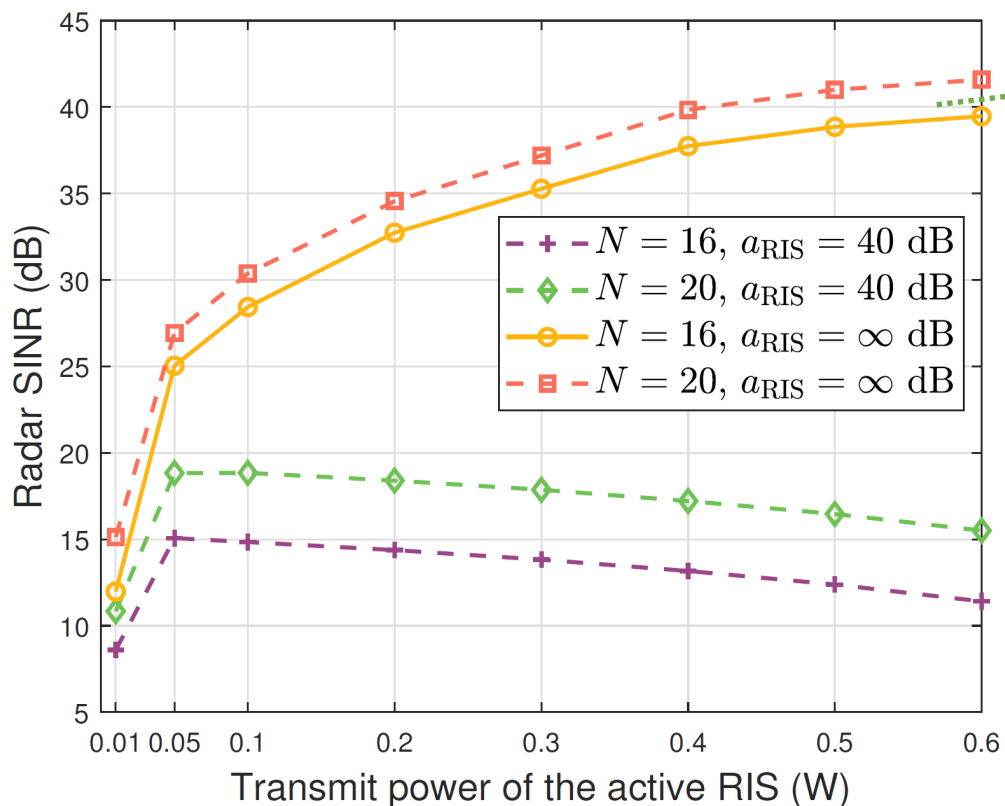


Fig. 4: Approximate radar SINR versus the the number of RIS elements N .

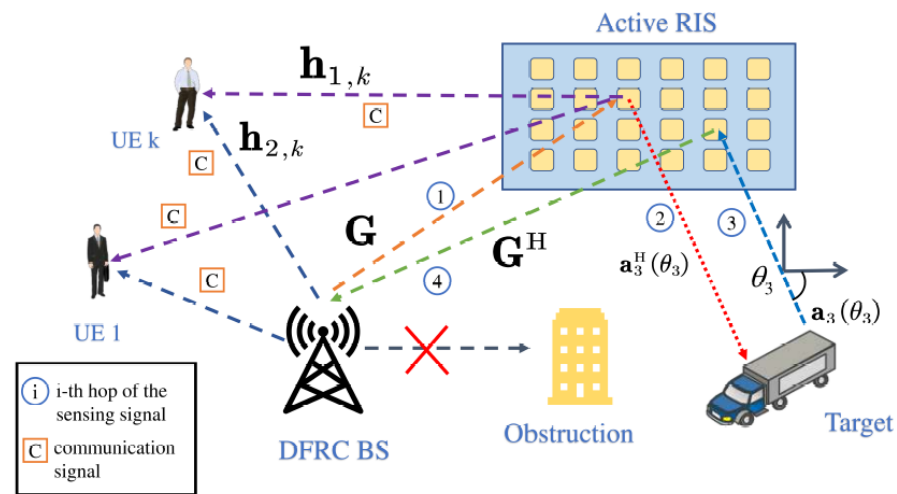
Transmit Power Allocation for a Moderate Number of Reflecting Elements



Lemma 6: If $P_t a_{RIS} N g^2 \geq C$, the optimal transmit power of the active RIS P_{RIS}^* satisfies $P_{RIS}^* \rightarrow C$. Otherwise, the optimal transmit power allocation should follow $(C - P_{RIS}^*) a_{RIS} N g^2 = P_{RIS}^*$.

No power amplification gain constraint scenario

It means that the active RIS can always amplify the received signal when the transmit power increases. In this scenario, it is advantageous to **allocate the power towards the active RIS compared to the BS.**



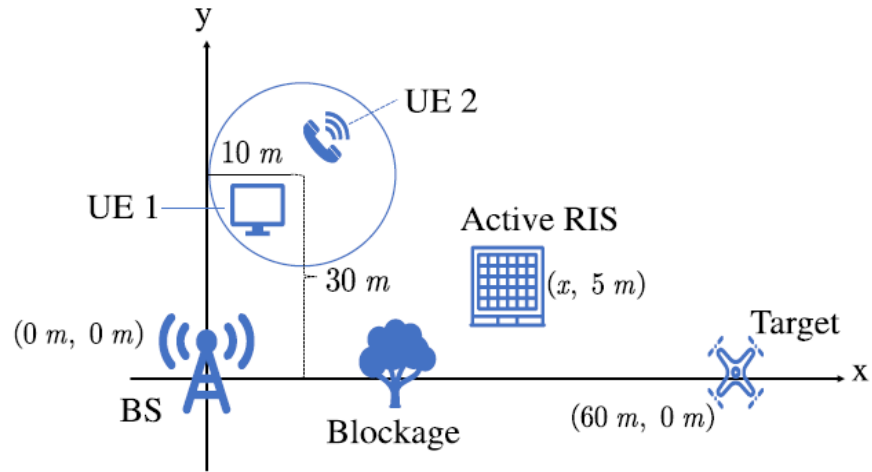
$$P_t \rho^4 N^3 \beta_r^2 g^2 \approx 10^{-6} P_t \rho^2 N g^2$$

The power of the second reflected signal

The power of the first reflected signal



Active RIS Deployment Strategy for a Moderate Number of Reflecting Elements



If $P_t a_{\text{RIS}} N g^2 \geq P_{\text{RIS}}$ it is better to deploy the active RIS near the target.

Otherwise, we have

$$\text{SNR} \triangleq \frac{P_t \rho^4 N^4 g^4 \beta_r^2}{\sigma_r^2}$$

$$\propto (d^2 + x^2)^{-\alpha_{\text{RIS}}} (d^2 + (D-x)^2)^{-\alpha_{\text{TG}}},$$

$$\propto x^{-2\alpha_{\text{RIS}}} ((D-x))^{-2\alpha_{\text{TG}}}, \text{ when } d \ll \min(x, D-x).$$

which means that we should either deploy the active RIS near the target or near the BS.

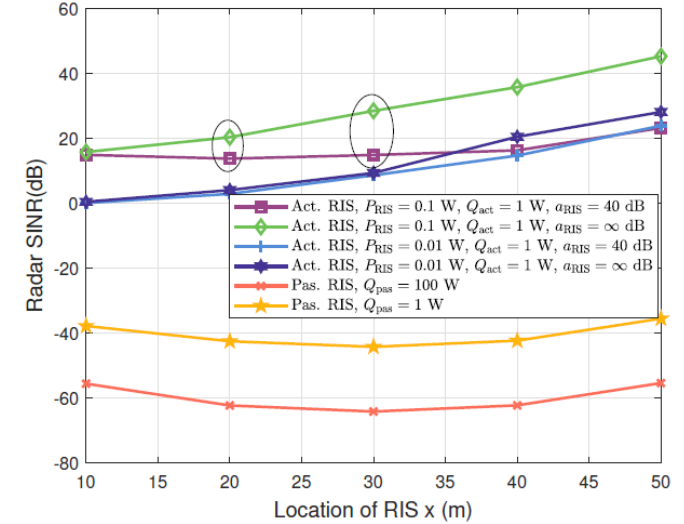


Fig. 7: Radar SINR versus the location of the RIS x .

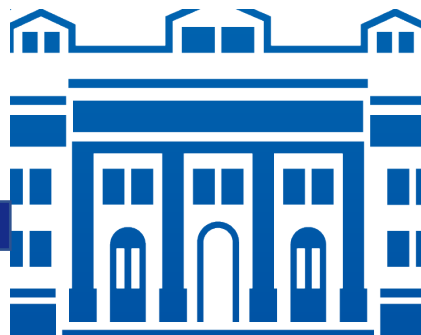
Conclusion:

- ❑ When the power amplification gain is tightly constrained, the radar SINR first decreases and then increases as x , which is similar to the findings in the passive RIS-aided communication systems [4].
- ❑ Put the active RIS near the Target is always a good solution.

4

Conclusions

Review and Outlook





Review & Outlook

Time	Core Concept	Significance
1948	Shannon Information Theory	Provide Guidance for Communication System Design
.....		
2018	RIS	From Adapting Environment to Changing Environment
2020	ISAC	Mutual Benefits in Communication and Sensing.
Nowadays	RIS meets ISAC	Leverage strengths and compensate for weaknesses.
Future	???	???



RISAC

Redefine the Intelligent Sensing and
Advanced Communication Network



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Active RIS Aided ISAC Systems: Beamforming Design and Performance Analysis

Thanks for your Listening!

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